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LAW OF RANDOM ERRORS.

BY R. J. ADCOCK, ROSEVILLE, ILLINOIS.

LET the unknown error in position of one measured determination of a point in space be δ , then the point sought may be anywhere on the surface of the sphere whose centre is the observed point and radius δ . Therefore, by the definition of probability, the probability that any point on this surface is the one sought is $\frac{1}{4\pi m \delta^2}$, where m is the number of points on a unit of surface.

Since then each random error gives a number of liable positions proportional to its square, therefore the probability that any one of the total number of liable points, resulting from n measured determinations of a point in space, is the one sought, is

$$y = \frac{1}{4\pi m S(\delta^2)},$$

where $S(\delta^2)$ is the sum of the squares of the random errors belonging to the n points given by measurement or observation. Hence of all points resulting from the n unknown errors, that which has the greatest probability is the one which makes $S(\delta^2)$ a maximum, that is, it is at the centre of gravity of the given points, and its probability is

$$p = \frac{1}{4\pi m S(\delta_1^2)},$$

where $S(\delta_1^2)$ is the sums of the squares of the distances from the given p'ts to their centre of gravity. Hence

$$y = p \frac{S(\delta_1^2)}{S(\delta^2)} = \frac{pS(\delta_1^2)}{S(\delta_1^2) + nx^2} = \frac{p}{1 + \frac{nx^2}{S(\delta_1^2)}},$$

where x , by problem 200 (*ANALYST*, Vol. V, page 91), is the distance from any one of the $4\pi m S(\delta^2)$ points to the centre of gravity of the n observed points, and y is the probability, frequency or density of errors at magnitude x . Hence

$$\int_0^x y dx = z = p \sqrt{\frac{S(\delta_1^2)}{n}} \tan^{-1} \sqrt{\frac{nx^2}{S(\delta_1^2)}},$$

where z is the number of errors included by x . Hence

$$n = p \sqrt{\frac{S(\delta_1^2)}{n}} \tan^{-1} \sqrt{\frac{nl^2}{S(\delta_1^2)}},$$

where l is the magnitude of the error which includes the whole number n .

$$\therefore \frac{z}{n} = \frac{\tan^{-1} \sqrt{\frac{nx^2}{S(\delta_1^2)}}}{\tan^{-1} \sqrt{\frac{nl^2}{S(\delta_1^2)}}} = \frac{\tan^{-1} \sqrt{\frac{nx^2}{S(\delta_1^2)}}}{2 \tan^{-1} \frac{1}{2}\sqrt{2}},$$

where $z \div n$ is the probability that an error shall not exceed x . It being proved at page 189, Vol. VII, that the constant

$$\tan \frac{1}{2} \tan^{-1} \left(\frac{nl^2}{S(\delta_1^2)} \right)^{\frac{1}{2}} = \frac{1}{2} \sqrt{2}.$$

Hence

$$x = \sqrt{\frac{S(\delta_1^2)}{n}} = \tan \left(\frac{2z}{n} \tan^{-1} \frac{1}{2}\sqrt{2} \right).$$

And for $z \div n = \frac{1}{2}$, the probable error is

$$x_1 = \frac{1}{2}\sqrt{2} \sqrt{\frac{S(\delta_1^2)}{n}}.$$

A line or surface is in its most probable position when the sum of the squares of the normals upon it, from the given positions, is a minimum, which normals are the errors.

In problem 239 (ANALYST, Vol. VI, p. 49), for A, $z \div n = \frac{36}{100}$, and his angular error $x = \tan^{-1} \frac{5}{3600} = \tan^{-1} \frac{1}{720}$. Hence

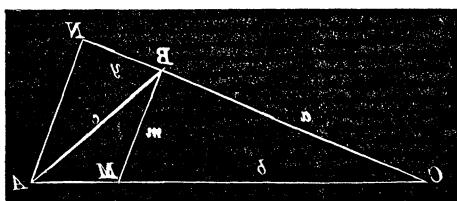
$$\frac{A's \text{ prob. er.}}{B's \text{ prob. er.}} = \sqrt{\frac{S(\delta_1^2)}{S(\delta_1'^2)}} = \frac{\tan \left(\frac{32}{25} \tan^{-1} \frac{1}{2}\sqrt{2} \right)}{\tan \left(\frac{18}{25} \tan^{-1} \frac{1}{2}\sqrt{2} \right)} \times \frac{\tan^{-1} \frac{1}{720}}{\tan^{-1} \frac{1}{540}} = 1.45.$$

SOLUTION OF A PROBLEM.

BY MARCUS BAKER, DIRECT'R OF U. S. MAGNETIC OBS., LOS ANGELES, CAL.

Problem:— In a plane triangle ABC there is given $a+b$, c and m , a perpendicular to BC drawn from B to AC , to solve the triangle.

Solution. From triangle ACN we have



$$b^2 = (a + y)^2 + c^2 - y^2 = a^2 + 2ay + c^2,$$

whence $2ay = b^2 - a^2 - c^2 = (a+b)^2 - 2a(a+b) - c^2$,

or $y = \frac{(a+b)^2 - c^2}{2a} - (a+b) = \frac{k}{a} - n$, (1)

where $2k = (a+b)^2 - c^2$ and $n = a+b$.